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BENDING RESPONSE OF FGM PLATE UNDER THERMAL AND THERMO MECHANICAL LOAD SUBJECT TO VARIOUS VOLUME FRACTION DISTRIBUTION LAWS

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Abstract

The functionally graded material (FGM) is one advanced composite whose property can be useful to accomplish the specific demands in various engineering applications. This is possible due to the material composition of the FGM which changes according to a law in a preferred direction. The thermo-mechanical analysis of FGM structures is one dimension which has attracted the attention of many researchers in the past few years in various engineering applications. It is found from the Thermo-mechanical. literature that Power law function, Sigmoid law function and Exponential law function have been used in various research works. In the current work, the estimation of properties of FGM and thereby it's modeling, has been carried out by considering all three FGM laws namely Power-law, Sigmoid and Exponential law functions with various values of volume fraction exponent and the non dimensional parameters have been compared.

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1. Introduction

The functionally graded material (FGM) is one advanced composite used to accomplish the specific demands in various engineering applications to achieve the advantage of the properties of individual material. This is possible due to the material composition of the FGMwhich changes according to a law. The thermo-mechanical analysis of FGM structures is one dimension which has attracted the attention of many researchers in the past few years in various engineering applications. Most researchers use the Power-law function (P-FGM), Sigmoid law function (S-FGM) or Exponential law function (E-FGM) to describe the volume fractions and thereby the effective variation in material properties through the thickness. Power-law function (P-FGM) has been used by Praveen [1], Marcelo [2] and Bhandari [3], Sigmoid law function (S-FGM) has been used by Shyang [4] and Bhandari [5] and Exponential law function (E-FGM) has been used by Bhavani^[6] and Hui^[7]. In the case of adding an FGM of Sigmoid law function to the multilayered composite that the Young's modulus changes are gradual because of using two Powerlaw functions together. The properties which are calculated using the Exponential law function are intermediate to those which are calculated using Power-law function (n=0.5) and Power-law function (n=2).Senthil [8] used the Laplace transformation technique to reduce equations governing the transient heat conduction to an ordinary differential equation (ODE) in the thickness coordinate which was solved by power series method. Cheng [9] and Reddy [10] obtained closed form analytical solution of a linear thermoelastic elliptic plate rigidly clamped at the mid-plane. Alshorbagyet. al. [11] concluded that FG plates provide a high ability to withstand thermal stresses, which reflects its ability to operate at elevated temperatures. The FGMs are more sensitive to the variation of the intensity of the heat flow, in or out of the structure, than that may be happened in the case of the isotropic material structures. Talha [12] reported that the temperature field and the gradient in the material properties have significant role on the thermomechanical behavior of the FGM plates. Sharma et.al. [13] examined the effect of material gradation profile on critical buckling parameters by evaluating the buckling response for a range of power law indexes. Xiaohui [14] considered transverse normal strain and found that the additional displacement parameters are not increased as transverse normal strain only includes the thermal expansion coefficient and thermal loading.

It is found from the literature that Power law function, Sigmoid law function and Exponential law function have been used in various research works. In the current work, the estimation of properties of FGM and thereby it's modeling, has been carried out by considering all three FGM laws namely Power-law, Sigmoid and Exponential law functions with various values of volume fraction exponent and the non dimensional parameters have been compared.

2. Methodology

The thermo-mechanical analysis is conducted for FGM made of Aluminum-Zirconia. The FGM plate is simply supported at all of its edges. The thickness of the plate (h) is taken 0.02m. The ratio of the plate side lengths is termed as aspect ratio (a/b). Thermal analysis was performed by applying temperature on the FGM plate. The ceramic top surface is subjected to a temperature of 100 °C. The metal surface and all the edges are kept at a temperature of 0 °C. The thermomechanical analysis has been performed by applying uniformly distributed load (udl) alongwith thermal load with varying aspect ratio (a/b). The value of udl (po) chosen was equal to $1X10^6$ N/m². The analysis is performed for various values of the volume fraction exponent (n) in P-FGM, S-FGM and E-FGM. The results are presented in terms of non-dimensional parameters i.e. non-dimensional deflection ($\overline{u_z}=u_z/h$), non-dimensional tensile stress ($\overline{\sigma_x}=\sigma_x/p_o$), non-dimensional shear stress ($\overline{\sigma_{xy}}=\sigma_{xy}/p_o$), Strain(e_x) and Shear strain (e_{xy}) where 'u_z' is deflection, ' σ ' is stress, 'a' and 'b' are side lengths of plate, and p_o is applied load (N/m²). Finite element method has been used to model the FGM plate in ANSYS.

3. Results

3.1 Effect of Aspect Ratio (A/B) in Constant Thermal Environment

In this study for FGM plate the comparison has been made for constant value of volume fraction exponent 'n'. The Fig. 1, 2, 3, 4 and 5 show the comparison graphs for various non dimensional parameters i.e. for pure ceramic (n=0), pure metal (n= ∞), P-FGM (n=2), P-FGM (n=0.5), S-FGM (n=2), S-FGM (n=0.5) and E-FGM.

The comparison of pure ceramic (n=0), pure metal (n= ∞), P-FGM (n=2), P-FGM (n=0.5), S-FGM (n=2), S-FGM (n=0.5) and E-FGM for square plate (aspect ratio a/b =1) reveals the following:

(a) The maximum non-dimensional deflection in case of P-FGM-n2 ($\overline{u_z} = 0.55$) is found to be greater than that of S-FGM-n2 ($\overline{u_z} = 0.36$). At the same time the non-dimensional deflection in case of P-FGM-n0.5 ($\overline{u_z} = 0.4$) is found to be more than that of S-FGM-n0.5 ($\overline{u_z} = 0.25$).

(b) The maximum non-dimensional tensile stress in case of P-FGM-n2 ($\overline{\sigma_x} = 43$) is found to be greater than that of S-FGM-n2 ($\overline{\sigma_x} = 40$). At the same time the non-dimensional tensile stress in case of P-FGM-n0.5 ($\overline{\sigma_x} = 28$) is found to be less than that of S-FGM-n0.5 ($\overline{\sigma_x} = 31$).

(c) The maximum non-dimensional shear stress in case of P-FGM-n2 ($\overline{\sigma_{xy}} = 510$) is found to be greater than that of S-FGM-n2 ($\overline{\sigma_{xy}} = 470$). At the same time the non-dimensional shear stress in case of P-FGM-n0.5 ($\overline{\sigma_{xy}} = 420$) is found to be less than that of S-FGM-n0.5 ($\overline{\sigma_{xy}} = 430$).



1) Non-Dimensional Deflection $(\overline{u_z})$

Fig. 1: Effect of aspect ratio (a/b) on non-dimensional deflection ($\overline{u_z}$) for simply supported plate under constant thermal environment for various FGM's, ceramic and metal



2) Non-Dimensional Tensile Stress $(\overline{\sigma_x})$

Fig. 2: Effect of aspect ratio (a/b) on non-dimensional tensile stress ($\overline{\sigma_x}$) for simply supported plate under constant thermal environment for various FGM's, ceramic and metal

3) Non-Dimensional Shear Stress $(\overline{\sigma_{xy}})$



Fig. 3: Effect of aspect ratio (a/b) on non-dimensional shear stress ($\overline{\sigma_{xy}}$) for simply supported plate under constant thermal environment for various FGM's, ceramic and metal





Fig.4: Effect of aspect ratio (a/b) on strain (e_x) for simply supported plateunder constant thermal environment for various FGM's, ceramic and metal



Fig.5: Effect of aspect ratio (a/b) on shear strain (e_{xy}) for simply supported plateunder constant thermal environment for various FGM's, ceramic and metal

(d) The maximum non-dimensional deflection for E-FGM ($\overline{u_z} = 0.51$) lies in between P-FGMn2 ($\overline{u_z} = 0.55$) and P-FGM-n0.5 ($\overline{u_z} = 0.4$). Similar trend is also observed in case of tensile stress, shear stress, strain and shear strain where the value of these parameters for E-FGM is found to be in between P-FGM n2 and P-FGM n0.5.

3.2 Effect of aspect ratio (a/b) in constant thermal environment under mechanical load

In this study for FGM plate the comparison has been made for constant value of volume fraction exponent 'n'. The Fig. 6,7,8,9 and 10 show the comparison graphs for various non dimensional parameters i.e. for pure ceramic (n=0), pure metal (n= ∞), P-FGM (n=2), P-FGM (n=0.5), S-FGM (n=2), S-FGM (n=0.5) and E-FGM.





Fig. 6: Effect of aspect ratio (a/b) on non-dimensional deflection ($\overline{u_z}$) for simply supported plate under constant thermo-mechanical load for various FGM's, ceramic and metal

2) Non-Dimensional Tensile Stress $(\overline{\sigma_x})$



Fig.7: Effect of aspect ratio (a/b) on non-dimensional tensile stress ($\overline{\sigma_x}$) for simply supported plate under constant thermo-mechanical load for various FGM's, ceramic and metal





Fig.8: Effect of aspect ratio (a/b) on non-dimensional shear stress ($\overline{\sigma_{xy}}$) for simply supported plate under constant thermo-mechanical load for various FGM's, ceramic and metal

4) Strain (ex)



Fig. 9: Effect of aspect ratio (a/b) on strain (e_x) for simply supported plate under constant thermo-mechanical load for various FGM's, ceramic and metal

5) Shear Strain (exy)



Fig.10: Effect of aspect ratio (a/b) on shear strain (e_{xy}) for simply supported plate under constant thermo-mechanical load for various FGM's, ceramic and metal

The comparison of pure ceramic (n=0), pure metal (n=∞), P-FGM (n=2), P-FGM (n=0.5), S-

FGM (n=2), S-FGM (n=0.5) and E-FGM reveals the following:

(a) The maximum non-dimensional deflection in case of P-FGM-n2 ($\overline{u_z} = 9.7$) is found to be greater than that of S-FGM-n2 ($\overline{u_z} = 9.3$). At the same time the non-dimensional deflection in case of P-FGM-n0.5 ($\overline{u_z} = 8$) is found to be less than that of S-FGM-n0.5 ($\overline{u_z} = 8.6$).

(b)The maximum non-dimensional tensile stress for square plate in case of P-FGM-n2 ($\overline{\sigma_x} = 534$) is found to be greater than that of S-FGM-n2 ($\overline{\sigma_x} = 500$). At the same time the non-dimensional tensile stress in case of P-FGM-n0.5 ($\overline{\sigma_x} = 438$) is found to be less than that of S-FGM-n0.5 ($\overline{\sigma_x} = 475$).

(c) The maximum non-dimensional shear stress in case of P-FGM-n2 ($\overline{\sigma_{xy}} = 860$) is found to be greater than that of S-FGM-n2 ($\overline{\sigma_{xy}} = 826$). At the same time the non-dimensional shear stress in case of P-FGM-n0.5 ($\overline{\sigma_{xy}} = 765$) is found to be less than that of S-FGM-n0.5 ($\overline{\sigma_{xy}} = 805$).

(d) The maximum strain (e_x) in case of P-FGM-n2 ($e_x = 0.0053$) is found to be greater than that of S-FGM-n2 ($e_x=0.005$). At the same time the strain (e_x) in case of P-FGM-n0.5 ($e_x = 0.004$) is found to be less than that of S-FGM-n0.5 ($e_x = 0.0043$).

(e) The maximum shear strain (e_{xy}) in case of P-FGM-n2 ($e_{xy} = 0.033$) is found to be greater than that of S-FGM-n2 ($e_{xy}=0.031$). At the same time the non-dimensional tensile stress in case of P-FGM-n0.5 ($e_{xy} = 0.025$) is found to be less than that of S-FGM-n0.5 ($e_{xy} = 0.027$).

(f) The maximum non-dimensional deflection for E-FGM ($\overline{u_z} = 9.1$) lies in between P-FGM-n2 ($\overline{u_z} = 9.7$) and P-FGM-n0.5 ($\overline{u_z} = 8.6$). Similar trend is also observed in case of other non-dimensional parameters.

3.3 Variable thermal environment

In this study for FGM plate the comparison has been made for constant value of volume fraction exponent 'n', aspect ratio (a/b=1) and at temperature 400°C. The Fig. 11, 12, 13, 14 and 15 show the comparison graphs for various non dimensional parameters i.e. for pure ceramic (n=0), pure metal (n= ∞), P-FGM (n=2), P-FGM (n=0.5), S-FGM (n=2), S-FGM (n=0.5) and E-FGM. The following observations are worthnoting:

1) Non-Dimensional Deflection $(\overline{u_z})$



Fig.11: Effect of variable thermal environment on non-dimensional deflection $(\overline{u_z})$ for simply supported square plate for various FGM's, ceramic and metal



2) Non-Dimensional Tensile Stress $(\overline{\sigma_x})$

Fig.12: Effect of variable thermal environment on non-dimensional tensile stress $(\overline{\sigma_x})$ for simply supported square plate for various FGM's, ceramic and metal





Fig.13: Effect of variable thermal environment on non-dimensional shear stress ($\overline{\sigma_{xy}}$) for simply supported square plate for various FGM's, ceramic and metal



4) Strain (ex)

Fig.14: Effect of variable thermal environment on Strain (e_x) for simply supported square plate for various FGM's, ceramic and metal

5) Shear Strain (exy)



Fig.15: Effect of variable thermal environment on Shear strain (e_{xy}) for simply supported square plate for various FGM's, ceramic and metal

(a) The non-dimensional deflection at 400°C, in case of P-FGM-n2 ($\overline{u_z} = 1.3$) is found to be greater than S-FGM-n2 ($\overline{u_z} = 0.8$). Also the non-dimensional deflection in case of P-FGM-n0.5 ($\overline{u_z} = 1$) is found to be greater than S-FGM-n0.5 ($\overline{u_z} = 0.6$).

(b) The non-dimensional tensile stress in case of P-FGM-n2 ($\overline{\sigma_x} = 175$) is found to be greater than S-FGM-n2 ($\overline{\sigma_x} = 160$). At the same time the non-dimensional tensile stress in case of P-FGM-n0.5 ($\overline{\sigma_x} = 115$) is found to be less than S-FGM-n0.5 ($\overline{\sigma_x} = 125$).

(c) The non-dimensional shear stress in case of P-FGM-n2 ($\overline{\sigma_{xy}} = 2100$) is found to be greater than S-FGM-n2 ($\overline{\sigma_{xy}} = 2050$). At the same time the non-dimensional shear stress in case of P-FGM-n0.5 ($\overline{\sigma_{xy}} = 1700$) is found to be less than S-FGM-n0.5 ($\overline{\sigma_{xy}} = 1800$).

(d) The strain (e_x) in case of P-FGM-n2 ($e_x = 0.0035$) is found to be greater than S-FGM-n2 ($e_x=0.0032$). At the same time the strain (e_x) in case of P-FGM-n0.5 ($e_x = 0.0015$) is found to be less than S-FGM-n0.5 ($e_x = 0.0027$).

(e) The shear strain (e_{xy}) in case of P-FGM-n2 ($e_{xy} = 0.075$) is found to be greater than S-FGM-n2 ($e_{xy}=0.07$). At the same time the non-dimensional tensile stress in case of P-FGM-n0.5 ($e_{xy} = 0.055$) is found to be less than S-FGM-n0.5 ($e_{xy} = 0.061$).

(f) The non-dimensional deflection for E-FGM ($\overline{u_z} = 1.25$) lies in between P-FGM-n2 ($\overline{u_z} = 1.3$) and P-FGM-n0.5 ($\overline{u_z} = 1$) and similar trend is also observed in other non-dimensional parameters. Further it is also observed that all non-dimensional parameters vary linearly with rise in temperature, though slope of all curves are found to be different.

3.4 Variable thermal environment under mechanical load

In this study for FGM plate the comparison has been made for various nondimensional parameters i.e.for constant value of volume fraction exponent 'n',for aspect ratio (a/b=1) and at temperature 400°C. The Fig.16,17,18,19 and 20 show the comparison graphs for pure ceramic (n=0), pure metal ($n=\infty$), P-FGM (n=2), P-FGM (n=0.5), S-FGM (n=2), S-FGM (n=0.5) and E-FGM.



1) Non-Dimensional Deflection $(\overline{u_z})$

Fig.16: Effect of variable thermal environment on non-dimensional deflection ($\overline{u_z}$) for simply supported plate under constant mechanical load for various FGM's, ceramic and metal



2) Non-Dimensional Tensile Stress $(\overline{\sigma_x})$

Fig.17: Effect of variable thermal environment on non-dimensional tensile stress ($\overline{\sigma_x}$) for simply supported plate under constant mechanical load for various FGM's, ceramic and metal

3) Non-Dimensional Shear Stress $(\overline{\sigma_{xy}})$



Fig.18: Effect of variable thermal environment on non-dimensional shear stress ($\overline{\sigma_{xy}}$) for simply supported plate under constant mechanical load for various FGM's, ceramic and metal





Fig.19: Effect of variable thermal environment on strain (e_x) for simply supported plate under constant mechanical load for various FGM's, ceramic and metal

5) Shear Strain (exy)



Fig.20: Effect of variable thermal environment on shear strain (e_{xy}) for simply supported plate under constant mechanical load for various FGM's, ceramic and metal

The following observations are worthnoting:

(a) The non-dimensional deflection in case of P-FGM-n2 ($\overline{u_z} = 4.2$) is found to be greater than S-FGM-n2 ($\overline{u_z} = 4.1$). At the same time the non-dimensional deflection in case of P-FGM-n0.5 ($\overline{u_z} = 3.3$) is found to be greater than S-FGM-n0.5 ($\overline{u_z} = 3.4$).

(b) The non-dimensional tensile stress in case of P-FGM-n2 ($\overline{\sigma_x} = 350$) is found to be greater than S-FGM-n2 ($\overline{\sigma_x} = 290$). At the same time the non-dimensional tensile stress in case of P-FGM-n0.5 ($\overline{\sigma_x} = 140$) is found to be less than S-FGM-n0.5 ($\overline{\sigma_x} = 210$).

(c) The non-dimensional shear stress in case of P-FGM-n2 ($\overline{\sigma_{xy}} = 2300$) is found to be greater than S-FGM-n2 ($\overline{\sigma_{xy}} = 2100$). At the same time the non-dimensional shear stress in case of P-FGM-n0.5 ($\overline{\sigma_{xy}} = 1700$) is found to be less than S-FGM-n0.5 ($\overline{\sigma_{xy}} = 1900$).

(d) The strain (e_x) in case of P-FGM-n2 ($e_x = 0.0048$) is found to be greater than S-FGM-n2 ($e_x=0.0042$). At the same time the strain (e_x) in case of P-FGM-n0.5 ($e_x = 0.0022$) is found to be less than S-FGM-n0.5 ($e_x = 0.003$).

(e) The shear strain (e_{xy}) in case of P-FGM-n2 $(e_{xy} = 0.08)$ is found to be greater than S-FGM-n2 $(e_{xy}=0.074)$. At the same time the non-dimensional tensile stress in case of P-FGM-n0.5 $(e_{xy} = 0.058)$ is found to be less than S-FGM-n0.5 $(e_{xy} = 0.065)$.

(f) The non-dimensional deflection for E-FGM ($\overline{u_z} = 4$) lies in between P-FGM-n2 ($\overline{u_z} = 4.2$) and P-FGM-n0.5 ($\overline{u_z} = 3.3$).

4. Conclusion

The behaviour of FGM plate under thermal and thermomechanical environment was studied. The work includes parametric study performed by varying volume fraction distribution and temperature. The close investigation of the various graphs for various non-dimensional parameters reveals the following information:

(a) The deflection values of FGM plates (i.e. $0 < n < \infty$) are much lower than that of fully metal plate. The non-dimensional tensile stress is minimum for metal and ceramic as compared to the FGMs. The non-dimensional tensile stress in the ceramic rich portion may be equal to that in the

metal rich region. As the value of volume fraction exponent increases the non-dimensional shear stress diverges and gives obvious trends.

(b)

(c)P-FGM (n=0.5) plate has the smallest deflection, strain and stress among all kinds of FGM plate. The reason is that the stiffness of the P-FGM (n=0.5) plate is more than that of E-FGM plate and stiffness of the E-FGM plate is more than that of P-FGM (n=2).FG plates provide a high ability to withstand thermal stresses, which reflects its ability to operate at elevated temperatures.The FGMs provide a highly stable response for the thermal loading comparing to that of the isotropic materials.

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